Ionization and boundary layer control.

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Abstract

The purpose of this paper is to investigate the possibility of preventing boundary layer energy losses by ionizing selectively the boundary layer air. Electrostatic repulsion at the wall surface of the positive ions created is the method chosen to prevent molecular collision at the surface and the resulting energy losses.

A subsonic diffuser section of deliberately poor design was the unit chosen as most suitable for testing the theory. The general method of attack was to attempt to show improved diffuser efficiency using an ionized air flow.

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Section I: The Mechanism of Ionization

Part A: The Bohr Theory of Extranuclear Atomic Structure

Before an adequate discussion of the phenomenon of ionization of gaseous atoms and molecules, that is the removal of one or more electrons from these bodies, can be presented, a clear picture of the laws which govern the actions of the orbital electrons is necessary. The commonly accepted theory which best describes the experimentally observed facts was that postulated by Bohr. Through the use of Planck's concept of the quanta of energy and by making two brilliant assumptions Bohr developed from theoretical considerations alone his theory which for simplicity will now be described only as it applies to the hydrogen atom. If one wishes to apply the powerful mathematics of wave mechanics, Bohr's theory can, however, be applied to any atomic structure.

If we picture the electron associated with a single atomic nucleus from the standpoint of classical mechanics alone, certain obviously incorrect conclusions can be drawn. It is a known fact that two unlike point charges such as the nucleus of the hydrogen atom and its electron attract each other according to Coulomb's inverse square law. Such a system can, therefore, not be in equilibrium unless there is relative motion. We can now make the following assumption and definitions:[1]

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1. Loeb, L. B. The Nature of a Gas, p.22. New York: John Wiley and Sons, Inc., 1931.

(1) The electron must be in motion about the nucleus and its orbit is either circular or elliptical.

(2) "e" = charge on proton or electron and is positive or negative respectively.

(3) Z = number of protons in nucleus.

(4) m = mass of electron.

(5) r = radius of orbit at any particular point.

(6) v = linear velocity of the electron.

The orbit may be established by writing:

$(Ze)\frac{e}{r^2}$ = attractive force between the electron and the nucleus.

$\frac{mv^2}{r}$ = centrifugal force on electron

Then:

[1] $$(Ze)\frac{e}{r^2} = \frac{mv^2}{r}$$

Now for any given values "Ze" and "m" it is obvious that there are an infinity of orbits defined by values of "v" and "r". It is further evident that an electron executing any such path must be continuously accelerating towards the nucleus i.e., it is under the action of a single central force. Maxwell's classical electrodynamics tells us that such an accelerated moving charge must radiate energy and its kinetic energy $\frac{1}{2}mv^2$ is continuously being used up as

electro-magnetic radiation causing a consequent decrease in potential energy. If this process continues to its logical conclusion the electron will follow a spiral path towards the nucleus and eventually fall into it, thus destroying the atomic configuration as we have visualized it. This we know cannot be the case for atoms are extremely stable and do not behave in this fashion.

Bohr solved the apparent paradox created by the classical concept of electron motion about the atomic nucleus by making several restricting assumptions based on Planck's quantum theory. He postulated that the centrally accelerated electron does not radiate as classical electrodynamics would lead one to believe. Considering equation (1) Bohr assumed further that "v" and "r" must be limited to values which define certain discrete orbits and not an infinity of orbits. The only possible orbits were assumed to be those in which the moment of momentum was some whole multiple of $\frac{h}{2\pi}$ where "h" is Planck's constant and equals 5.64×10^{-27} erg secs. [2] [3]

A comparatively simple development of the restriction which Bohr places on the angular momentum is possible. If the orbit of the electron is assumed to be circular, it is easily shown that the mean value of the kinetic energy of the electron taken for a whole

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2. Planck, M. "Zur Theorie der Wärmestrahlung", Vol. 31, pp.758-768 (1910) and "Über die Begründung des Gesetzes der Schwarzen Strahlung". Vol. 37, pp. 642-656 (1912), Annalen Der Physik.

3. Andrade, E. N. Structure of the Atom, 3rd Edition p. 184. New York: Harcourt Brace and Co., 1927.

revolution is equal to "E". [4] Here "E" is the amount of energy which must be transferred to the system to remove the electron to an infinitely great distance from the nucleus.

The essence of Planck's derivation of his universal constant "h" is that the energy radiation in an atomic system does not take place in a continuous way as described by classical electrodynamics.[5] On the contrary it takes place in distinctly separate emissions and the amount of energy radiated out from an atomic vibrator is equal to "nhu" where "n" is some integer and "u" is the characteristic frequency of the atomic vibrator. In the case of the electron executing its orbit about the atomic nucleus if "w" is the frequency of revolution then:

$$u = \frac{w}{2} \qquad \text{where} \qquad w = \frac{v}{2\pi r}$$

Let us now assume that we have an electron at a great distance from an atomic nucleus and with no velocity with respect to it. Then, if we allow this electron to approach the nucleus and take a stable orbit, during the binding of the electron we would expect by Planck's theory to have a radiation of energy equal to "nhu". Then the energy involved in this change of state must be:

$$E = \frac{nhw}{2} = nhu$$

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4. Bohr, N. "On the Constitution of Atoms and Molecules." Philosophical Magazine, Vol. 26, p.3 (1913).

5. See Planck, M. op. cit.

If the angular momentum is denoted by "M", for a circular orbit:

$$\pi M = \frac{K.\ E.}{w}$$

But for a circular orbit:

$$K.\ E. = e = \frac{nhw}{2}$$

Then:

$$M = n M_0$$

Where M_0 is the unit of angular momentum and:

$$M_0 = \frac{h}{2\pi}$$

which demonstrates the validity of Bohr's assumption.[6]

We can write the expression for kinetic energy of the electron from (1) as:

$$(2) \qquad K.E. = \frac{1}{2} mv^2 = \frac{Z e^2}{2r}$$

Now since moment of momentum is some even multiple of $\frac{h}{2\pi}$ if "n" is any integer:

$$(3) \qquad mvr = \frac{nh}{2\pi}$$

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6. Bohr, N., op. cit., p. 15.

and the electron can exist only in orbits defined by this expression. The discrete values of "r" allowable may now be found.

From (3):

$$(4) \qquad v^2 = \frac{n^2h^2}{(2\pi mr)^2}$$

From (2)

$$(5) \qquad \frac{mn^2h^2}{2(2\pi mr)^2} = \frac{Ze^2}{2r}$$

$$(6) \qquad r = \frac{n^2h^2}{4\pi^2mZe^2}$$

Equation (6) then defines the radii of the permissible electron orbits according to the Bohr theory. These may be evaluated by giving "n" the values 1, 2, 3, 4, etc.

From Coulomb's Law the potential energy of the electron is:

$$(7) \qquad P.E. = \frac{-Ze^2}{r}$$

Then the total energy of the electron is:

$$(8) \qquad E = K.E. + P.E. = \frac{Ze^2}{2r} - \frac{Ze^2}{r} = \frac{-Ze^2}{2r}$$

Substituting (6) into (8):

$$(9) \qquad E = \frac{-2\pi^2mz^2e^4}{h^2}\frac{1}{n^2}$$

Equation (9) defines the definite energy levels which may exist and these are the only energy levels which may be expected to be stable.

Bohr further assumed that monochromatic radiation is brought about when an electron jumps from one stable state of higher energy content E_2 to a lower energy state E_1 and that:

$$(10) \quad E_2 - E_1 = hu$$

where "u" is the frequency of the electro-magnetic wave or light emitted. Substituting for E_1 and E_2:[7]

$$(11) \quad u = \frac{2\pi^2 mz^2 e^4}{h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

This equation is, however, correct only for a stationary nucleus. So if we now recognize the fact that both nucleus and electron must be in motion about a common mass center the final expression becomes:

$$(12) \quad u = \frac{2\pi^2 mz^2 e^4}{\left(1 + \frac{m}{M}\right)h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where "M" is the mass of the nucleus.[8]

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7. Bohr, N., op. cit., pp. 1-25.

8. Stranathan, J. D. The Particles of Modern Physics, pp. 215-224. Philadelphia: The Blakiston Co., 1942.

These concepts are all indispensable to a clear understanding of the mechanism of ionization. The conclusions drawn from them have been substantiated by a vast number of experiments and all show that Bohr has given us a mathematical description of the phenomenon which is in very close agreement with the physical facts.

Part B: Ionization and Excitation Processes

The general picture of the extranuclear structure of the atom involves energy relations between electrical charges. In a particular atom or molecule, say of oxygen, the electrons are arranged in certain definite energy levels as defined by Bohr. Each electron, however, in the natural undisturbed state has taken the orbit of lowest energy at which it can stably exist. If we could supply by some means the exact quanta of energy as defined in Part A needed for an electron to go from one energy level to another there is no reason to believe that it would not do so. Or if the energy could be supplied for the removal of an electron from the influence of the nucleus we might expect as a result an atom or molecule with a positive charge. It has been experimentally proven that both of these phenomena can and do occur. In the first case when an electron has been moved to a higher energy level by the absorption of energy from some outside source the atom or molecule is said to be "excited"; in the latter when an electron has been completely removed it becomes positively charged and is said to be "ionized".

By experiment it has been found that a number of elementary particles can transfer to the orbital electrons the energy needed for ionization or excitation. A few of the many possible means which are of importance to this discussion are listed here:[9] [10]

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9. Wheatcroft, E. L. E. Gaseous Electrical Conductors, pp. 23-26. Oxford: The Clarendon Press. 1938.

10. See Loeb, L. B., op.cit., p. 128

1. Rapidly moving electrons including beta rays.
2. Rapidly moving positive charges, protons and alpha particles.
3. Rapidly moving positive ions in high fields.
4. Photoelectric ionization by ultra-violet light, X-rays and gamma rays.
5. Thermal ionization by collision of neutral molecules or atoms at high temperatures.

One of the best means for studying the excitation and ionization potentials, i.e. the quantities of energy necessary to excite or ionize, in a gas is to use accelerated electrons as the source of energy. In the simplest form the experimental technique is to seal a quantity of the gas to be studied in a glass tube containing two electrodes and a heated filament to serve as a source of electrons. The arrangement is indicated in figure one.* The electrons are emitted from the heated filament F and given an acceleration or an amount of kinetic energy:

$$\frac{1}{2} mv^2 = Ve$$

where V is the potential difference between F and the wire mesh anode G and e is the charge on the electron. A retarding potential V_R is applied between G and P nearly equal to V say 0.5 volts less, thus no

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* See figure one on the following page

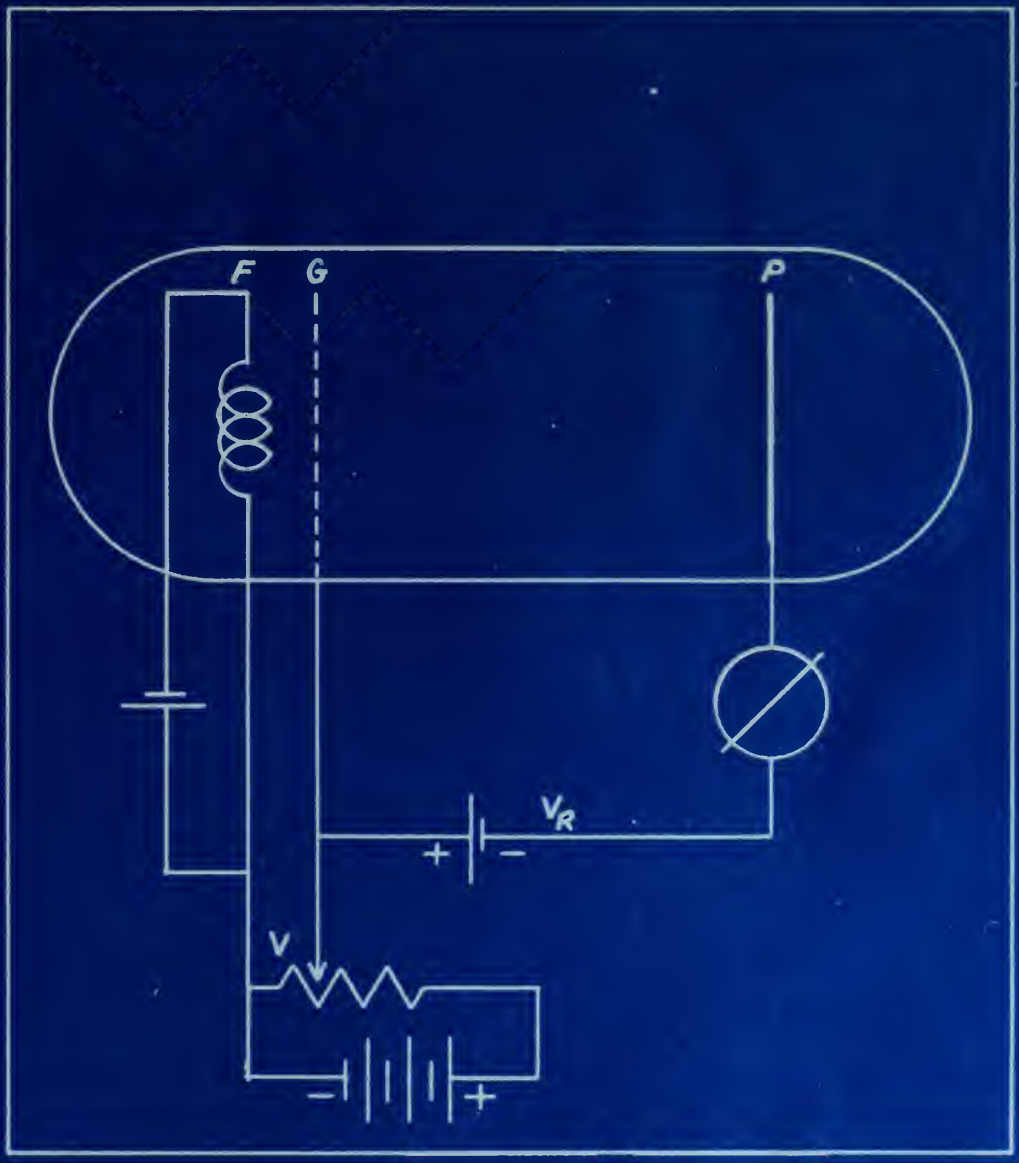

FIGURE 1
APPARATUS FOR DETERMINING EXCITATION AND IONIZATION POTENTIALS

additional kinetic energy is added between the anode and cathode. The gas pressure and tube element spacing is such that comparatively few collisions of the electrons with gas molecules occur between F and G and the order of magnitude of the distance from G to P is one mean free path.[11]

If the gas being studied is for example molecular nitrogen (N_2) and a plot is made of the voltage V vs. the current which reaches P a steady rise in current is noted for voltages below 8.5. In other words the electrons which pass G headed toward the cathode with their initial energy Ve which do collide with the gas molecules must do so elastically and, therefore, lose no energy and eventually arrive at P. When the accelerating voltage reaches 8.5 volts, however, a distinct drop in the rate of current increase is noted. Some of the electrons must have suffered inelastic collisions, i.e. given up some or all of their initial kinetic energy to the gaseous molecules. According to Bohr's theory this indicates that the electrons have acquired an energy Ve equal to hu, the quantum required to raise an electron of the N_2 molecule to the next higher energy level. This value of V equal to 8.5 volts is then an excitation potential. If V is still further increased a second excitation potential is noted at 9.5 volts and finally at 16.7 volts the ionization potential is reached and the accelerated electrons have attained sufficient energy to remove an orbital

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11. For a more complete discussion see Stranathan, op. cit., pp. 229-236.

electron from the molecule.[12]

Very precise experimental data obtained by investigators such as Compton and VanVoorhis indicates that the process although it obeys the laws derived by Bohr is not as clear cut as the foregoing would seem to lead one to believe.[13] It has been found that every electron does not ionize or excite a molecule or atom as soon as it has the requisite energy. Curves of the probability of ionization by electron impact as a function of velocity, expressed in equivalent volts, after Compton and VanVoorhis, for a large number of different gases shows that the maximum probability occurs in almost all cases between 100 and 200 equivalent volts. These findings are also borne out by Wheatcroft and Loeb.[14] [15]

The actual probability that ionization will occur, however, varies quite widely. The lowest value reported by Compton and VanVoorhis was about 0.21 for helium and the highest was for argon at 0.47 while N_2 discussed above showed a maximum probability of 0.42 at 175 equivalent volts. In any attempt to induce the maximum possible ionization in a gas in the shortest possible time these facts are all important. At voltages below the 100 to 200 required for the maximum

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12. Wheatcroft, E. L. E., op.cit., p.30. Experimental data on ionization and excitation potentials of various gases.

13. Compton, K. T. and VanVoorhis, C. C. "Probability of Ionization of Gas Molecules by Electron Impacts." Physical Review, Vol. 26, pp. 436-53, (1925).

14. Wheatcroft, E. L. E., op.cit., Chap.1.

15. Loeb, L. B., op. cit., p.111.

number of ions the probability of ionization falls precipitously to zero at the critical ionization potential and at voltages above this range the number of ions produced falls off exponentially.

Since an ionized or excited atom or molecule is at a higher energy level than is required for stability it will eventually return to its normal or lower energy level. In general it may be said that the life of positive ions in a gas is quite long, of the order of seconds in many cases, while the duration of an excited state is usually very short of the order of 10^{-8} seconds.[16] As is to be expected when ionized particles capture an electron or in the case of the excited atom or molecule when an electron jumps to a lower energy level an amount of energy equal to hu is radiated at a frequency u as defined by Planck and Bohr. Spectroscopic examination of the radiation from a large number of gases in the ionized and excited states by Saha and others has shown that Planck and Bohr predicted this phenomenon with amazing accuracy.[17]

Although the use of electrons as the means of conferring the energy of ionization or excitation lends itself particularly well to quantitative experimentation, it is by no means the only method possible. Photoelectric ionization by light of the proper frequency will cause ionization in a gas and the use of ultra-violet light for

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16. Ibid., p.115.

17. Saha, M. N., and Saha, N. K. A Treatise on Modern Physics, Vol. 1, p.674. Allahabad and Calcutta: The Indian Press, 1934.

such studies in air has been frequently tried.[18] High velocity particles emitted from radioactive substances especially alpha particles are a powerful ionizing agent.[19] X-rays have been used for direct study of the phenomena and as an indirect aid in many historic cases such as Millikan's oil drop experiment. Difficulties encountered in devising equipment have for the most part, however, led investigators to the use of the electron as the best means of conferring energy in a controlled manner on the gas being studied.

After the first initial ionization has taken place, regardless of the manner in which it has been brought about many secondary processes are brought into play. These may include thermal ionization by rapidly moving neutral atoms or molecules, if there has been a temperature rise, or photoelectric ionization of the gas due to photons emitted by excited or ionized particles within it. Positive or negative ions or electrons created by the original ionization may also cause additional energy changes. Unfortunately the quantitative effects and the relative importance of these various secondary phenomena is still a subject of uncertainty. A case in point is that of ionization by positive ions. Townsend and many prominent scientists who support his theory believe that ionization by positive ions is an important factor in the overall picture of the phenomena, while Loeb and anequally large group of

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18. Hughes and DuBridge. Photoelectric Phenomena, pp. 273-280. New York: McGraw Hill, 1932.

19. See Stranathan, op. cit., p.336.

Part C: The Mean Free Path Concept

If the method of ionization chosen involves the acceleration of the particle from which the energy is to be obtained in an electrostatic or electromagnetic field the mean free path in the gas under study must be known. The importance of this factor can be easily seen by recalling that the gaseous atoms or molecules may be either excited or ionized depending on the energy of the colliding particle. Further, the probability that they will be ionized is greatest at some particular energy level. Now if the field through which the ionizing particles are being accelerated is such that in one free path they cannot acquire the ionizing velocity their energy may well be employed merely to excite the atoms or molecules with which they come into contact. There is also the necessity for giving the accelerated particles not only the ionizing energy but sufficient additional energy so that they will ionize in the fewest number of collisions if the time to ionize is an important factor.

The idea of a mean free path was first introduced by the mathematical physicist Clausius in 1857 in support of Joule's theory of the kinetic nature of a gas and to describe the motion of gaseous molecules. The original calculations are rigorous and unwieldy but it is possible to present a simpler although equally valid proof of the theory.[22]

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22. Loeb, L. B. The Nature of a Gas, pp. 63-71. New York: John Wiley and Sons, Inc., 1951.

In nature the path of a gaseous molecule twists and turns in a random manner as it suffers successive collisions. However, let us imagine that the path of this molecule is straight and along the axis of a cylinder. The size of the cylinder we can arbitrarily choose and, therefore, the following definitions can be made:

σ = radius of the cylinder

$\frac{\sigma}{2}$ = radius of the molecules

c = molecular velocity in cm/sec and length of cylinder in cm.

N = molecules per cubic centimeter

The volume of the cylinder is $\pi\sigma^2 c$ cm^3 and on the average the total number of molecules contained in it is $\pi\sigma^2 cN$. Therefore, the total number of impacts sustained by a molecule moving along the longitudinal axis of the cylinder is $\pi\sigma^2 cN$. The average distance between impacts is:

$$L = \frac{c}{(\pi\sigma^2 Nc)} = \frac{1}{\pi\sigma^2 N}$$

All molecules in a gas do not, however, move with the same velocity and they are in relative motion with respect to one another. Maxwell by correcting for the velocity distribution and relative motion has developed the following expression for the mean

free path.[23]

$$L = \frac{1}{\sqrt{2}\,\pi\delta^2 N}$$

When ionization is brought about by electron collisions as in the case of corona discharge it is important to know what energy can be imparted to the electron during one free path by the electric field in which it is moving. This knowledge will enable the experimenter to apply the correct field strength for maximum ionization in minimum time i.e. the fewest number of collisions.

Consider the velocity of an electron relative to that of a singly ionized H_2 molecule which is the lightest and therefore fastest moving molecule. If both traverse the same potential difference in an applied field we can equate the kinetic energies and solve for the velocity ratio.[24]

$$\frac{1}{2} m_e c_e^2 = \frac{1}{2} m_{H_2} c_{H_2}^2$$

$$m_e c_e^2 = m_{H_2} c_{H_2}^2$$

$$c_e = c_{H_2} \sqrt{\frac{m_{H_2}}{m_e}}$$

$$c_e = c_{H_2} \sqrt{3680}$$

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23. Ibid. p.66.

24. Loeb, L. B. Fundamental Processes of Electrical Discharge in Gases, p. 645. New York: John Wiley and Sons, Inc., 1939.

Since the velocity of the electron is $\sqrt{3680}$ times that of the fastest moving ionic molecule all gaseous molecules can be assumed at rest relative to an electron moving among them. The $\sqrt{2}$ term appearing in the expression for the mean free path L derived by Maxwell is to compensate for the fact that the molecules are in relative motion but in the case of the electron it has been shown that the relative motion is negligible and therefore it will drop out.

The radius of each individual molecule has been defined as $\frac{\sigma}{2}$ but since each molecular center can approach every other center within a distance σ before impact occurs, we can define σ as the collision radius. It has been shown by experiment that the collision radius of the electron is $\frac{\sigma}{2}$, then:[25]

$$\sigma_e = \frac{\sigma}{2}$$

$$\sigma = 2\sigma_e$$

Substituting in Maxwell's mean free path equation and solving for L_e:

$$L_e = \frac{4\sqrt{2}}{\pi\sqrt{2}(2\sigma_e)^2 N} = \frac{4\sqrt{2}}{\pi\sqrt{2}\,4\,\sigma_e^2 N} = \frac{1}{\pi\,\sigma_e^2 N}$$

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25. Ibid. p. 646.

This expression developed from kinetic theory describes the mean free path of the electron as though it acted like a point charge. Actually electrons by virtue of their electric field and de Broglie wavelength do not interact like point charges with molecules. Very fast electrons may pass fairly close to molecules without deflection while slow moving electrons are badly deflected and collision is much more likely. This fact explains the extremely low probability of ionization at high electron energies previously discussed. R. B. Brode has experimentally determined the probability of ionization by electron collision at various electron energies and his data furnish a means of determining the proper potential drop per electron mean free path for maximum ionization.[26]

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26. Brode, R. B. "The Quantitative Study of the Collisions of Electrons with Atoms." Review of Modern Physics, Vol. 5, pp. 257-279 (1933).

Part D: Ionization By Negative Point Corona Discharge

The corona type of electrical discharge has long been familiar to mariners as St. Elmo's fire. It has, however, been only in the last few decades that the commercial necessity for long distance transmission of high voltages initiated investigations into the exact nature of the phenomena. As a result of these experiments we can now generally define a corona discharge in a gas as a localized breakdown of the gas in the vicinity of the conductor from a relatively good insulator to a conducting medium brought about by high field strengths in regions of comparatively small radii of curvature.[27]

If a gas contained only neutral molecules or atoms and there were no positive or negative ions or free electrons present it would be a perfect insulating medium until something happened to change its nature. In the case of air under standard conditions there exist a number of free electrons and positive and negative ions produced from natural causes. Cosmic rays alone are responsible for the formation of 1.6 ion pairs per cubic centimeter per second.[28] Ultra-violet light from the sun produces many more by photoelectric ionization, and natural radioactivity in the earth's crust plus ionization by collision of neutral atoms or molecules results in a few more. It is easy then to see that although air may be considered a good insulator it is not

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27. Loeb and Meek, op. cit., p.163.

28. See Stranathan, op. cit., p. 489.

a non-conductor and in fact, if something can be done to increase the number of charged particles in it, it may become an excellent conductor.

In the case of a negative point to plane corona in air at atmospheric pressure the characteristic blue glow which is always present is caused by intense ionization and excitation in the region between the positive and negative electrodes. The first step in the development of the corona is the emission of at least one electron from the negative point. There are two ways in which this can occur. If the field were high enough there would be spontaneous ejection of electrons from the surface of the conductor, sometimes called high field emission. Before the field has reached this intensity, however, in air another factor comes into play. A positive ion already existing for reasons stated above will be accelerated toward the negative electrode and upon striking it will if the potential difference through which it has fallen has been high enough, cause ejection from the surface of one or more free electrons. These electrons once free of the negative electrode will be accelerated away from it toward the anode.

What happens to these free electrons as they travel through the gas depends on several previously discussed concepts. First of all it is easy to deduce from the Bohr theory that if they can attain the ionizing energy hu necessary to detach an orbital electron from a molecule or atom on collision with it, the result of the first inelastic impact will be two electrons and one positive ion.

In order to obtain this energy the field strength must be such that during one free path the electron will reach the ionization velocity or some greater speed. If the field strength is too low for this to occur, the electron may either suffer successive elastic collisions until it reaches the anode or it may merely expend its energy in inelastic collisions resulting in excitation of the particles which it contacts. If maximum ionization is the desired result the field strength must be more than simply that required to impart the critical velocity to the electron. It must be such that during one free path the electron will fall through the potential difference required to yield the maximum probability of ionization in one impact.

J. S. Townsend in one of the earliest studies of ionization by collision developed the expression for the number of ions produced for a given distance from the origin by the initial ionizing particle.[29] Although he assumed that the negative ionizing particles were primarily negative ions we now know that they are electrons. His theory, however, can still be applied to the problem at hand to determine the number of positive ions formed by one or more electrons originating at the negative point. Assume the following:[30]

α = number of ion pairs formed by electron collision per centimeter of travel

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29. Townsend, J. S. Electricity in Gases, pp. 260-336. Oxford: The Clarendon Press, 1915.

30. See Stranathan, op. cit., p. 12.

x = distance from cathode

dx = increment of distance

n = number of electrons originating at the cathode

n_x = number of ion pairs formed at a distance "x" from the cathode

dn = number of new pairs of ions formed in distance dx

Then, remembering that each time an electron ionizes an atom or molecule at least two electrons and one ion are the result:

$$dn = n\alpha\, dx$$

Integrating for the total ions formed:

$$\int_1^{n_x} \frac{dn}{n} = \int_0^x \alpha\, dx$$

$$\left[\log n\right]_1^{n_x} = \left[\alpha x\right]_0^x$$

$$n_x = \mathcal{E}^{\alpha x}$$

Examination of this equation reveals at once that even one electron originating at the negative point and traveling in a field sufficiently intense to impart to it the ionizing velocity will quickly produce a shower of positive ions and newly liberated electrons. Such a shower is commonly referred to as an electron avalanche.

With these facts at hand the sequence of events in the corona discharge from a negative point can be stated. As soon as the field intensity at some point on the surface of the negative point reaches a value high enough to permit an incoming positive ion to supply enough energy to overcome the work function of the cathode material, a free electron will be ejected. Further, if the field intensity is sufficient to impart the ionizing velocity to this electron, an electron avalanche will result and the rate of increase of free electrons and positive ions will follow Townsend's equation as developed above.[31]

If the corona discharge takes place in a gas under relatively high pressure, say air at atmospheric pressure, the mean free path is of the order of 10^{-5} centimeters. This results in the formation of large numbers of positive ions in a very short distance from the cathode itself. Since the electronic mass is only $\frac{1}{3680}$ that of the lightest molecule, it must inevitably have a much greater mobility that the positive ions of nitrogen and oxygen formed in air. We can, therefore visualize the free electrons produced by the multiple ionization taking place as rapidly moving charges leaving behind them a dense cloud of relatively immobile positive ions.

Before the formation of this cloud of positive ions an electron in the vicinity of the cathode would be very strongly

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31. Loeb, L. B. Fundamental Processes of Electrical Discharge in Gases, p.515. New York: John Wiley and Sons, Inc., 1939.

repelled. Now as these electrons recede from the negative point leaving behind them their cloud of positive ions they are in effect building up a space charge which very quickly becomes strong enough to shield them from the repelling cathode. This results in a slowing down of the free electrons.[32]

Another factor which further inhibits the recession of the free electrons from the negative point is the decreasing field strength. For an infinitely small point all lines of force would converge on an infinitely small area and at the surface of such a point the repulsive force acting on the electron would be infinite. In actual practice, of course, this is not possible but if the diameter of the point used is of the order of say one thousandth of an inch, it will certainly be very great; the exact magnitude depending only on the applied potential between the cathode and the anode. As the electron recedes from the cathode, however, the lines of force diverge sharply and the total applied repulsive force is correspondingly decreased and the electron slows down.

Since it has been shown that there are two factors both tending to slow down the electron as it recedes from the negative point, it is obvious that there will be a point at which it no longer has sufficient energy to ionize. At this stage two things may happen to these slow moving electrons. They may, if the gas

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32. Ibid.

used is air, be immediately captured by a neutral oxygen molecule and form a negative ion or they may suffer a series of inelastic collisions causing excitation only of the gas and then form negative oxygen ions. It has been shown experimentally that only infrequently do they form negative ions with nitrogen.

If the gap between electrodes is so short and the pressure so low that the electron avalanche is able to reach the anode before the mechanism described above can stop its progress, a spark and not a corona type discharge will of course result. In the case of corona discharge where the streamer has terminated in the space between the electrodes, only the characteristic blue glow is observed.[33]

The foregoing has in effect described one cycle only, of the corona discharge. Although the first streamer is stopped, the cloud of positive ions adjacent to the cathode remain. Very quickly, one of these positive ions or perhaps another existing in the air due to ultra-violet, cosmic ray or natural radioactivity ionization, will plunge to the surface of the cathode freeing new electrons. During the time taken for this to occur there has been some diffusion of the space charge and a new electron avalanche can take place producing

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33. See Loeb and Meek, op. cit. "Theory of Streamer Formation."

new positive ions.[34] [35] [36] [37]

The fact that there was such a natural frequency of corona discharge was first noted and measured by Trichel in 1938.[38] [39] Trichel using various geometric arrangements of his electrodes and various field strengths found the natural discharge frequency to vary from 5000 cycles to at least 200,000 cycles per second which was the limit of the resolving power of the oscillograph which he used. Loeb, however, points out that at the very high frequencies, there are many small areas emitting electron avalanches alternately; exactly which one depending upon the instantaneous value of the field strength which, as shown above, may be profoundly influenced by heavy positive ion space charges.[40]

See the following authors for a general description of the phenomena of corona discharge:

34. Loeb, L. B., op. cit., pp. 514-535.

35. Farwell, S. P. "Brush Discharge Phenomena." Physical Review, Vol. 4, pp. 31-39 (1914).

36. Mackenzie, D. "The Corona in Air." Physical Review, Vol. 5, pp. 294-310 (1915).

37. Crooker, S. J. "Direct Current Corona From Different Surfaces and Metals." Physical Review, Vol. 8, pp. 344-363, (1916).

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38. Trichel, G. W. "The Mechanism of Negative Point to Plane Corona." Physical Review, Vol. 54, pp. 1078-1084 (1938).

39. Trichel, G. W. "The Mechanism of Positive Point to Plane Corona." Physical Review, Vol. 55, pp. 382-390. (1939).

40. Loeb, L. B. op. cit., p. 617.

Although ionization by electron collision is the chief factor in the formation of corona about electrical conductors, there are several other mechanisms operating both to increase and diminish the total number of ions present. Since the exact importance of each is not as yet fully understood by investigators in the field, they will only be metioned here. Ionization by positive ion collision was early proposed by J. S. Townsend as possible mechanism, but today it is still a matter of controversy and is questioned by many prominent physicists.[41] Photo-electric ionization due to photons emitted by excited molecules or atoms is known to take place in the corona but to what extent has not yet been measured. Negative-ion collision is known to produce some ionization but here again, we lack a quantitative determination in the corona, although J. S. Townsend has defined the mechanism well.[42] Lastly there is a certain amount of recombination of the positive ions in the gas which under carefully controlled conditions as an isolated phenomenon can be accurately measured but which has not been accurately determined for the corona discharge.

Much work remains to be done before the mechanism of the corona discharge can be explained with certainty on a quantitative basis. We now have an understanding of the mechanisms involved

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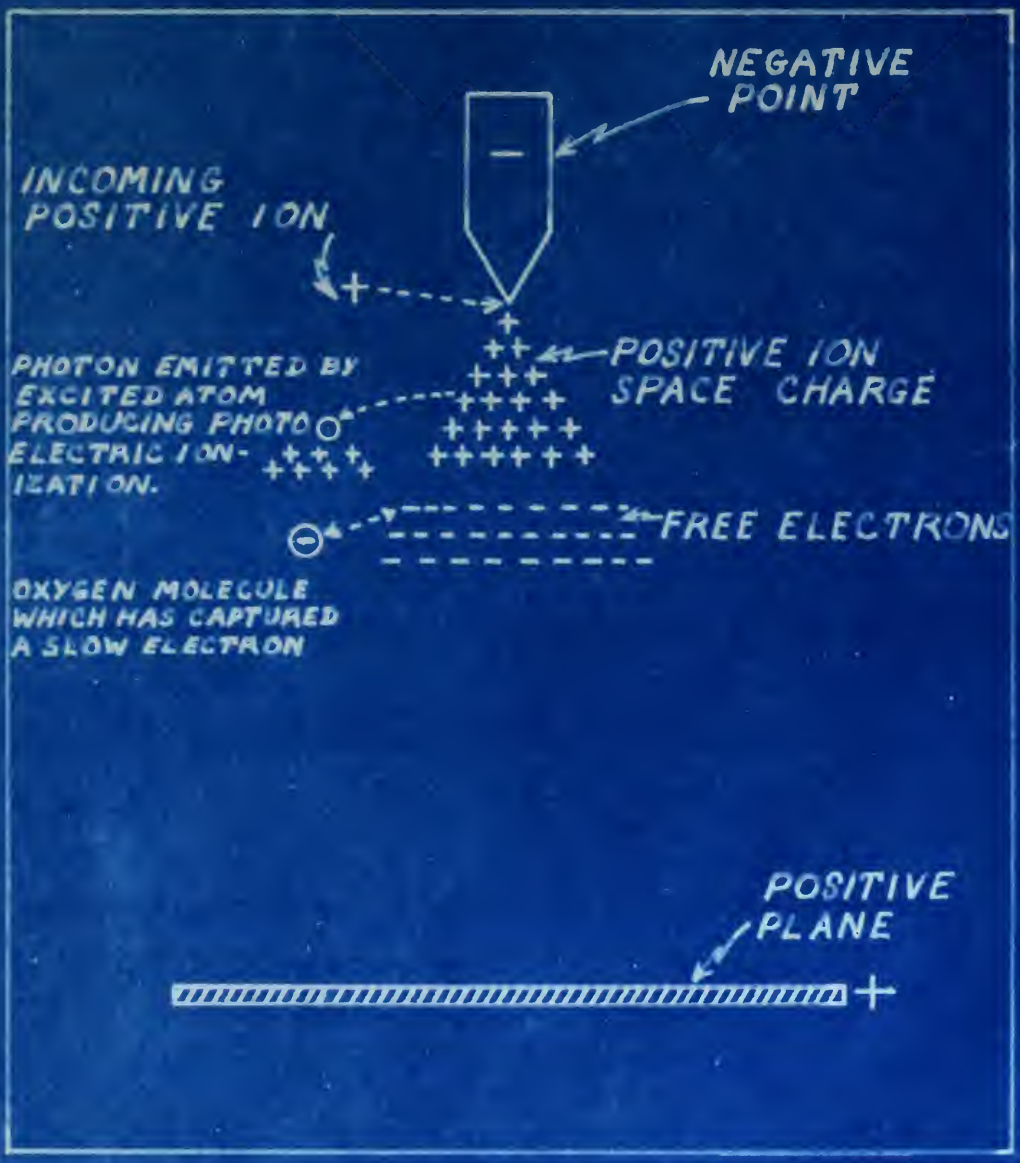
41. Stranathan, op. cit., pp. 18-19.

42. Townsend, J. S. "The Genesis of Ions by the Motion of Positive Ions in a Gas." Philosophical Magazine, Vol. 6, pp. 598-618 (1903).

but there remains the task of discovering the exact importance of each. Figure two portrays the various mechanisms discussed above and attempts to show them operating in their proper sequence through one cycle of the phenomenon as we now understand it.



SCHEMATIC REPRESENTATION OF THE CORONA DISCHARGE

FIGURE 2

In a natural air flow subject only to the forces of friction and the temperature or pressure gradients responsible for its velocity, a boundary layer forms at the walls confining the flow which has been adequately described by Von Karman and others. Although most of the work in the field of the thermodynamics of air flow has regarded the gaseous medium as a continuum for purposes of quantitative mathematical analysis, there are many advantages to a qualitative examination from the kinetic or atomic viewpoint. In fact the energy losses arising from wall frictional effects may be completely described on this basis.

If we visualize an individual molecule moving adjacent to the wall its entire motion may be decomposed into four distinct velocity vectors. As a result of the temperature at which the medium exists and by the law of equipartition of energy, the individual molecule will have a translational velocity due solely to its thermal energy which may be decomposed into three equal velocity vectors directed along the X, Y, and Z axes of a three dimensional orthogonal coordinate system. The fourth component of the molecular motion is that due to the stream velocity and it may be represented by a free vector equal in magnitude and direction to the average stream velocity at this point. It is this directed component of motion which has been produced only after the expenditure of energy to create the necessary pressure or temperature gradient which is lost when the molecule collides with the wall.

Unlike a light ray which upon striking a mirror is reflected at a well defined angle without energy loss, a molecule which strikes a surface is temporarily adsorbed with the resultant loss of its entire stream velocity. At some later time it is ejected into the stream at a velocity which is a function of the wall temperature. Thus along a wall surface we can visualize the frictional effect as arising from a large number of slow, low energy molecules ejected from the surface undergoing collisions with other molecules which have not yet contacted the wall and robbing them of all or part of their stream velocity. This effect is continuously communicated outward and is responsible for the velocity distribution in the boundary layer which varies from zero at the wall to the stream velocity at the outer limit.

The fundamental proposition of this thesis is that if the molecules of the flowing air stream can be prevented from striking the walls confining the stream that there will be no energy losses due to the effects described above. If it were possible to prevent all the molecules from striking the wall there would, indeed, be no boundary layer.

Physically this effect is being sought by attempting to saturate the boundary layer with large numbers of positive ions of nitrogen and oxygen. The entire flow is then placed in an electrostatic field so directed that the positive ions will everywhere be repelled from the surface and thus prevented from striking it and giving up their stream velocity component.

Part A: Air Flow Equipment

The vehicle chosen as lending itself most readily to a test of the effect of boundary layer control by ionization in a flowing air stream was a subsonic diffuser of deliberately poor design. The details of the particular units used are illustrated in figures 3,4,5,6, and 7. The diffuser itself was designed with an area ratio of two to one and a six degree angle on each wall. This combination assures a poor pressure rise efficiency which it is the object of this investigation to attempt to improve. The metering nozzle and the accelerating nozzle section attached to the diffuser itself were both designed in accordance with A.S.M.E. specifications.

Instrumentation of the air flow was accomplished by mercury and water manometers. Micro-manometers were used in locations where the pressures to be measured were sufficiently small to require them. Qualitative data on the air flow was also provided for by small probes with fine threads attached for visual observation of flow characteristics.

The air metering chamber immediately downstream from the metering nozzle was designed to provide as steady and laminar flow as possible for the diffuser section under test. The length of the chamber used was thirty inches and it was equipped with screen baffles to further smooth out the flow.

Air supply was obtained from a fifty horsepower Schramm reciprocating air compressor. This compressor is rated at 200 c.f.m.

TITLE
SCHEMATIC DRAWING OF FLOW METERING APPARATUS, DIFFUSER AND ELECTRICAL CONNECTIONS.

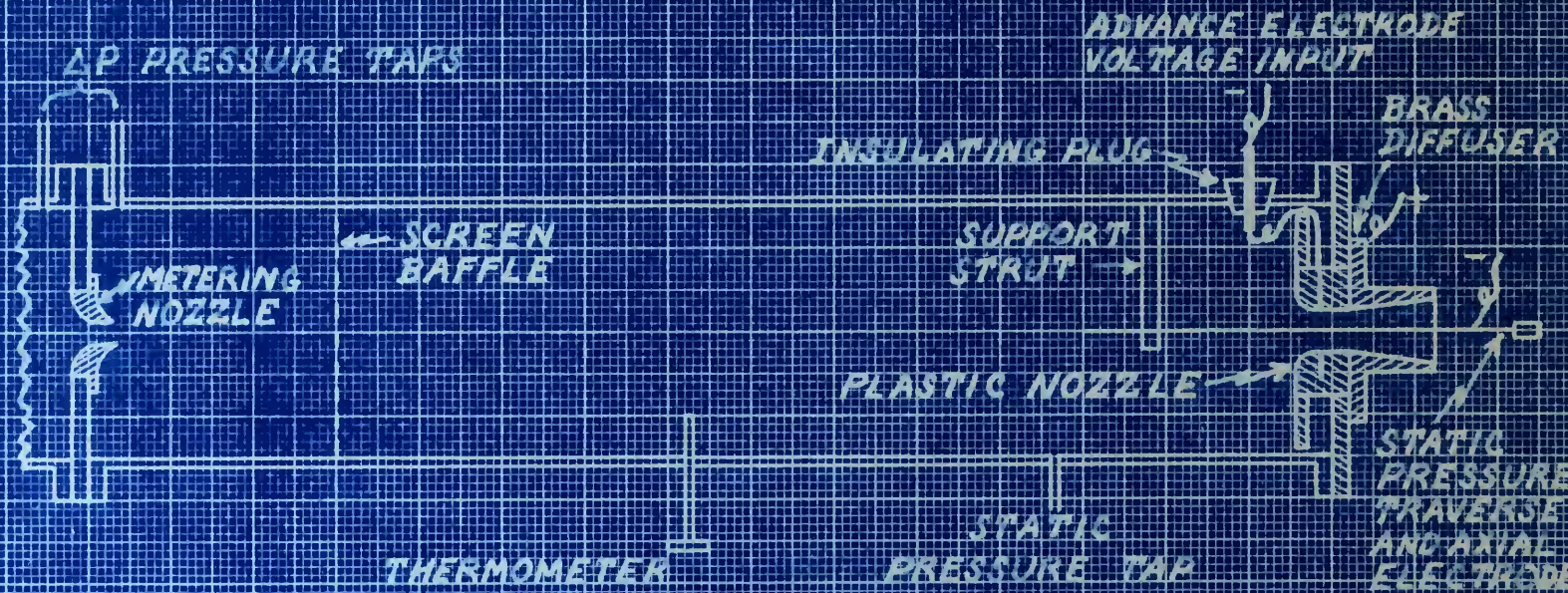
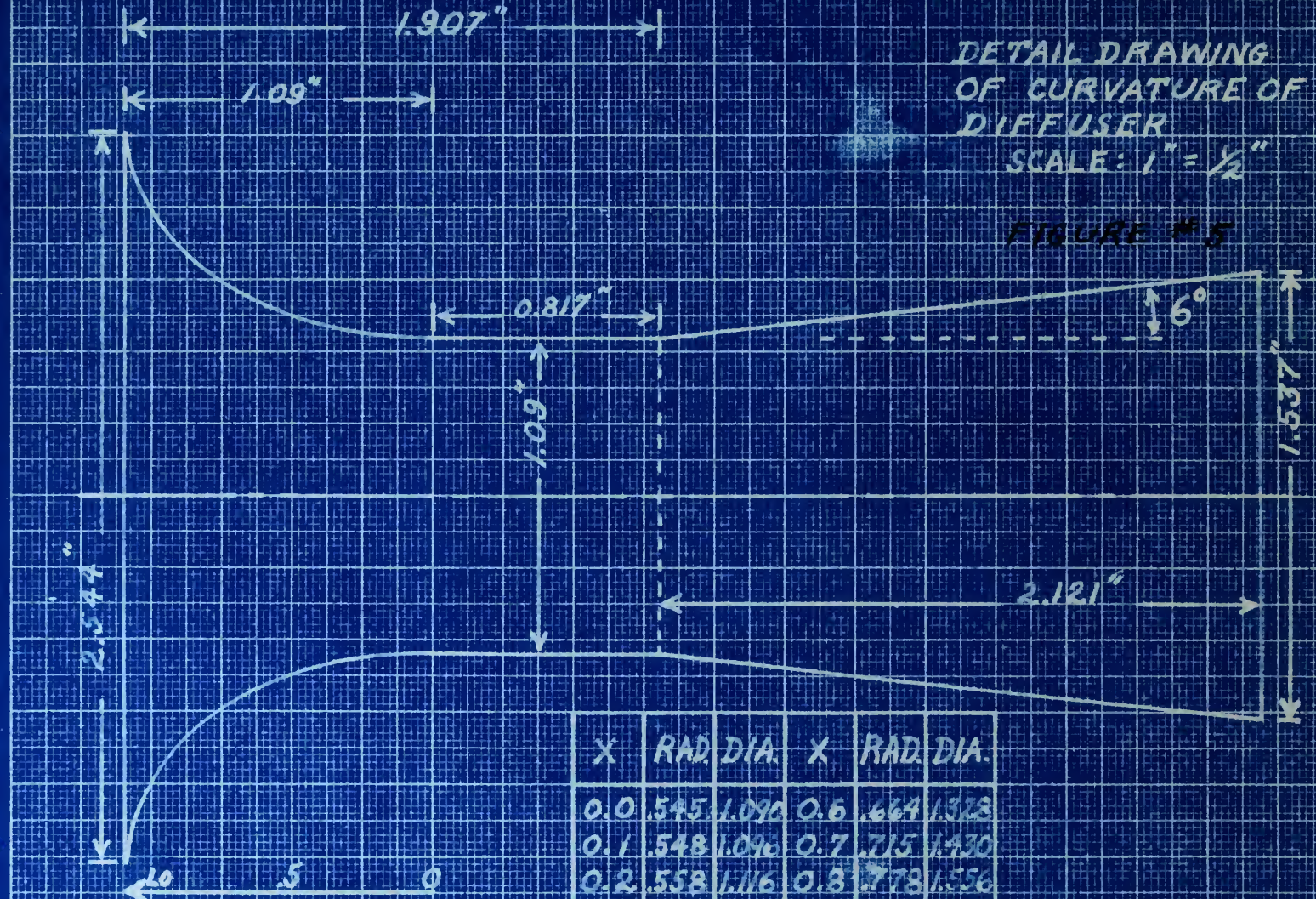


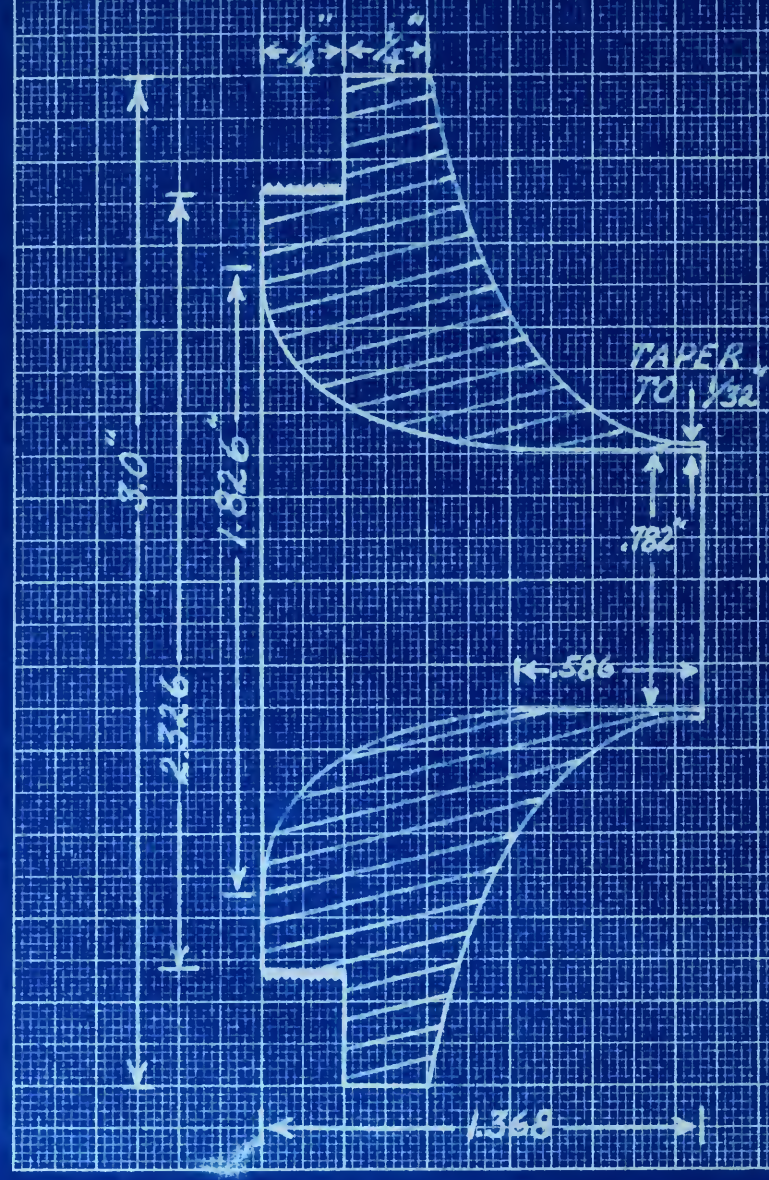
FIGURE #3



DETAIL DRAWING OF CURVATURE OF DIFFUSER
SCALE: 1" = ½"

FIGURE # 5

X	RAD.	DIA.	X	RAD.	DIA.
0.0	.545	1.090	0.6	.664	1.328
0.1	.548	1.096	0.7	.715	1.430
0.2	.558	1.116	0.8	.778	1.556
0.3	.573	1.146	0.9	.861	1.722
0.4	.596	1.192	1.09	1.272	2.544
0.5	.626	1.252			



METERING NOZZLE

THE NOZZLE IS TO BE THREADED AS SHOWN SO THAT IT MAY BE SCREWED INTO THE CENTER OF A ¼" PLATE WHICH HAS BEEN DRILLED TO FIT A STANDARD 6" PIPE FLANGE. THE ENTRANCE TO THE NOZZLE SHOULD BE FLUSH WITH THE SURFACE OF THE PLATE.

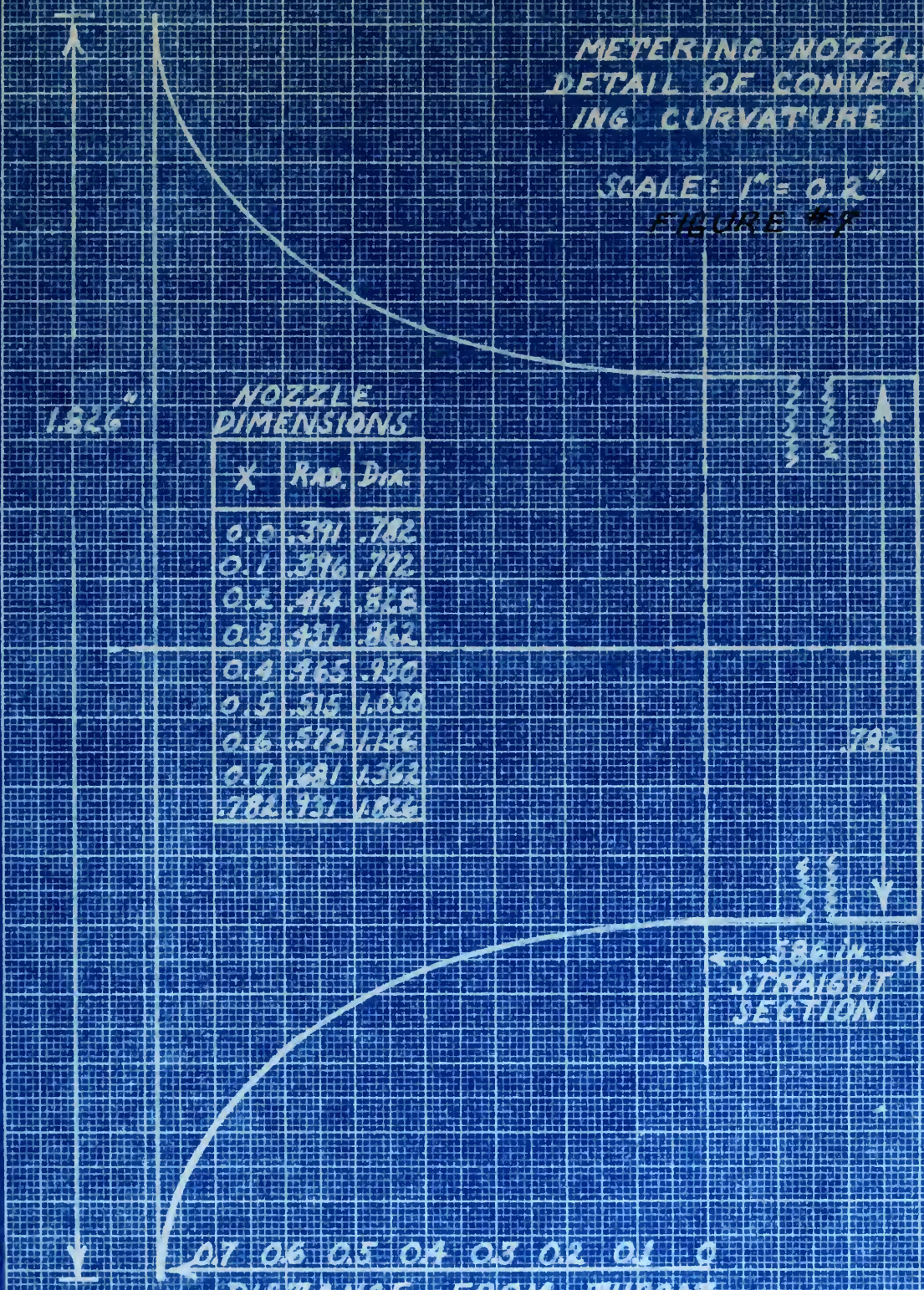
SCALE: 1" = ½"

FIGURE #2

METERING NOZZLE DETAIL OF CONVERGING CURVATURE

SCALE: 1" = 0.2"

FIGURE #1



NOZZLE DIMENSIONS

X	RAD.	DIA.
0.0	.391	.782
0.1	.396	.792
0.2	.414	.828
0.3	.431	.862
0.4	.465	.930
0.5	.515	1.030
0.6	.578	1.156
0.7	.681	1.362
.782	.913	1.826

at the inlet. However, to provide a safe working margin all designs were based on a maximum weight flow corresponding to 175 c.f.m. at the compressor inlet at 530° R total temperature.

Part B: Electrical Equipment

The electrodes for ionisation of the boundary layer air and for providing the necessary electrostatic control field within the diffuser are the key electrical units. Since the boundary layer is the region which it is desired to ionize, the advance electrodes have been imbedded circumferentially in the wall of the straight section of the plexiglass nozzle leading to the brass diffuser. These electrodes consist of one thousandth inch shim brass sandwiched in between the one fourth inch plexiglass sheet of which the nozzle was constructed. The upstream electode is charged negatively and the downstream electrode is one fourth of an inch upstream from the diffuser entrance electrically connected to the diffuser and both are grounded. The object of this arrangement is to cause a corona type discharge between the sharp edges of the shim brass electrodes through the boundary layer only of the flowing air stream just before it enters the diffuser. This will result in a large supply of positive ions flowing in the boundary layer of the diffuser.

A static pressure search tube of one tenth inch outside diameter steel tubing is inserted in the center of the diffuser throat and supported well upstream in the metering chamber by two plexiglass insulating struts and outside the diffuser by a plexiglass insulator. This tube has been carefully centered and serves both as a centerline static pressure tap and as the negative electrode of the electrostatic control field in the diffuser.

By charging the search tube negatively with respect to the wall of the diffuser the positive ions entering the diffuser boundary layer will tend to migrate toward the centerline negative electrode. By adjusting the potential difference between the positive diffuser wall and the negative search tube the velocity of positive ion migration may be chosen which yields the greatest net increase in diffuser pressure rise efficiency or the most desirable velocity profile at any cross section.

Two high voltage low current supplies were used. One was employed to produce the corona discharge potential between the two advance electrodes. The other was employed to furnish the electrostatic field in the diffuser. If proper voltage dividing equipment had been available one supply would have sufficed for both tasks.

The 100 kilovolt supply used for the corona discharge was designed and built by the General Electric Company. It consists essentially of a step-up transformer capable of 100 K. V. output on the secondary with a 440 volt input to the primary. Two kenotron tubes each acting as a half wave rectifier convert the sixty cycle input to a direct current output with about ten percent ripple at full load. Control equipment as shown in figure 8 is provided to give voltage outputs varying from zero to full load.

The 50 kilovolt supply for the diffuser electrostatic field was designed and built by Mr. A. deGraffenried of the Aeronautical Engineering Department at Rensselaer Polytechnic Institute.

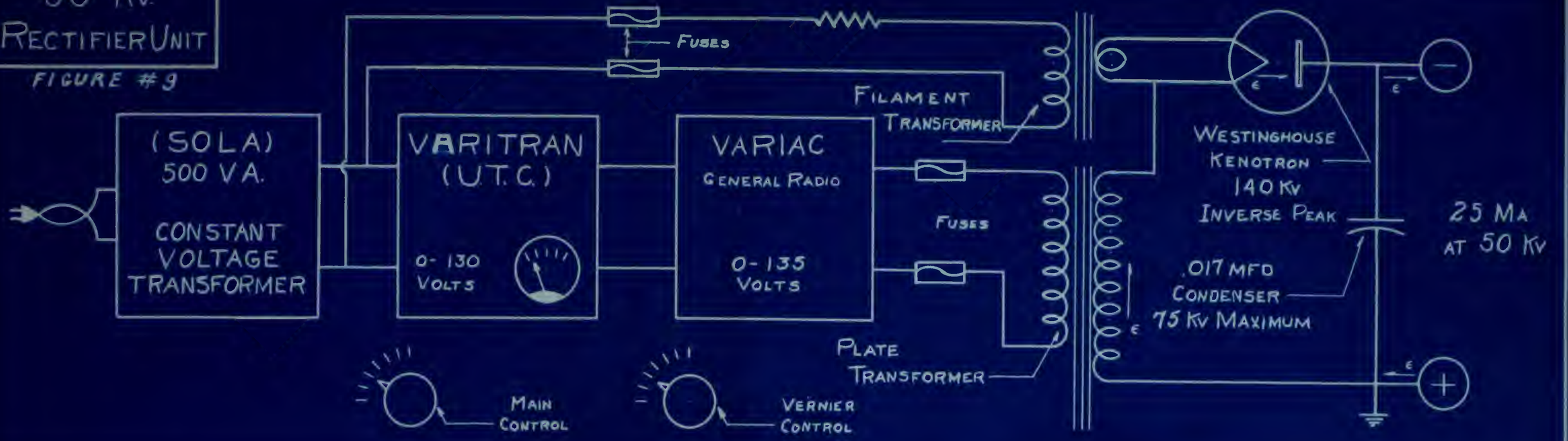
Description of Equipment (37)

It consists of a Westinghouse Kenotron tube and associated equipment as shown in figure 9. This particular supply was chosen for control of the electrostatic field because it has provisions for accurate control and measurement of the applied potential which are lacking in the larger unit.

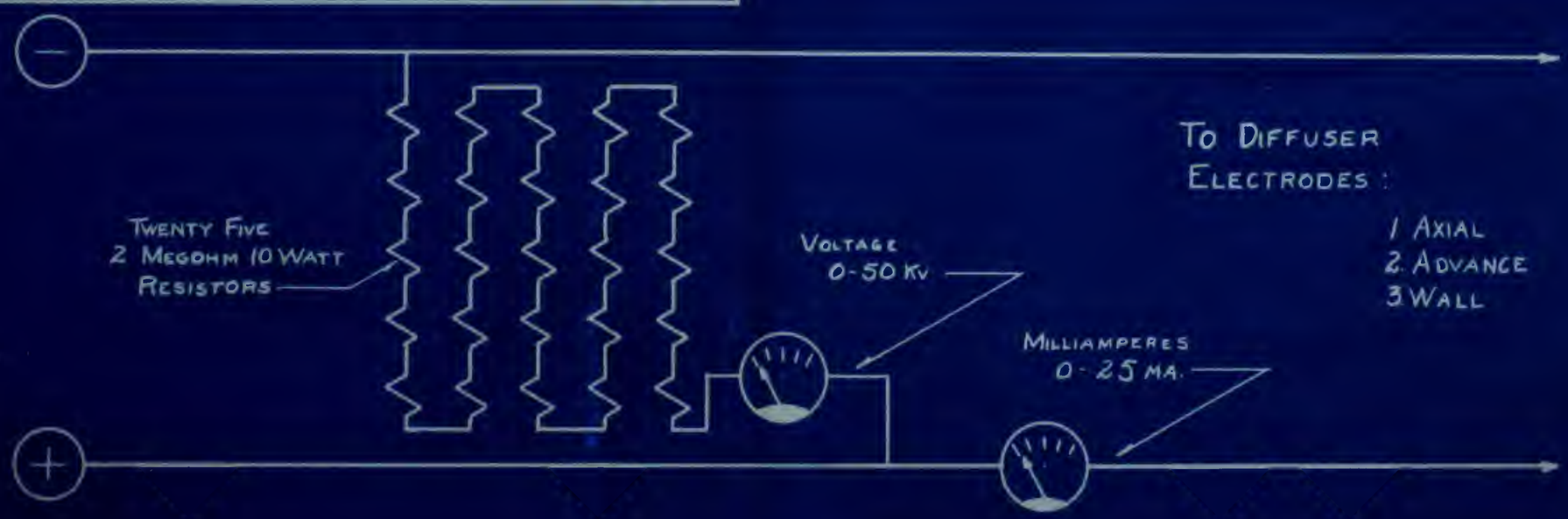
FIGURE #8
30 MA
100 KV D.C.
KENOTRON
TESTING SET

50 Kv. Rectifier Unit

FIGURE #9



Voltage and Current Measuring Circuits



Preliminary plans called for increasing the weight flow at which separation would occur in the diffuser by ionizing the boundary layer. This was regarded as a valid criterion for the effective reduction of boundary layer energy losses for it is these very losses which cause the separation to occur. The quantitative effects would be measured as an increase in diffuser pressure rise efficiency and energy efficiency.

The first problem to be solved was to determine just exactly when separation did occur. The usual methods of static pressure traverse or visual search of the exit plane boundary layer with a probe and fine thread could not be used. With no axial electrostatic field and no ionization the separation point could be easily found by the above methods, however, once the electrical fields necessary for ion control and corona discharge were established no pressure readings taken inside the diffuser could be relied upon. False readings were attributed to ion bombardment of the static pressure openings due to gradients in field strength at the sharp edges of the pressure taps. The probe method was also unusable because the thread acting as a visual guide would pick up a charge in the form of attached ions and tend to align itself with the electrostatic field rather than with the air flow.

An attempt was then made to define separation by use of pressure readings taken at the inlet and outlet of the diffuser where the effects of the electric fields used were comparatively small. This was done by operating the diffuser at various weight flows and

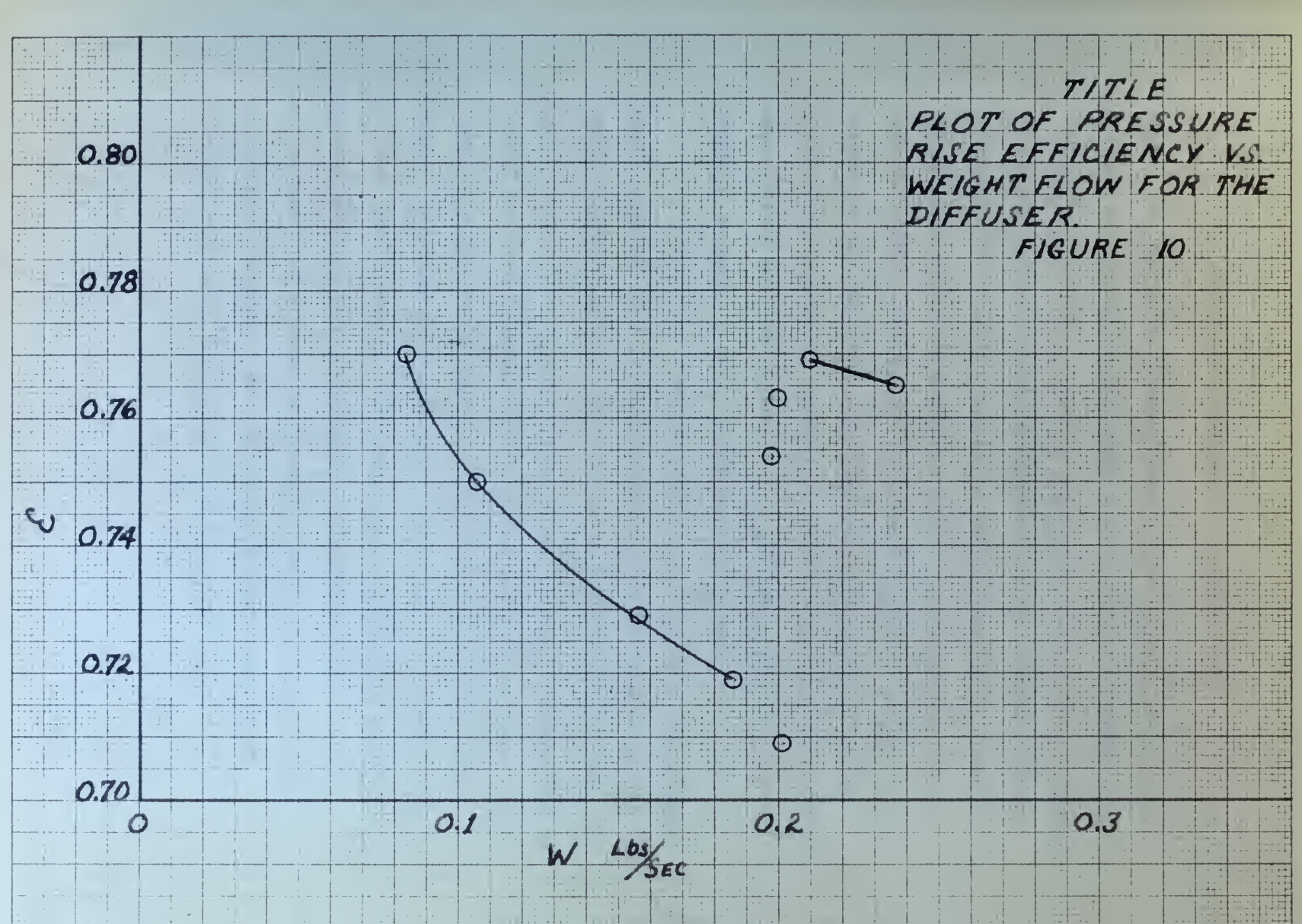
noting inlet and exit mach numbers and pressures. The calculated diffuser efficiency was then plotted against weight flow as shown in figure 10.

It can be seen from this curve that an apparent increase in diffuser efficiency has occurred at a weight flow of about 0.19 pounds per second. This of course represents the point or rather the beginning of the range of separation. The calculated rise in efficiency shown at this point is entirely the result of the fact that the diffuser is being given credit for discharging to atmospheric pressure from its throat mach number and pressure. In reality separation has occurred at some point inside the diffuser and the actual area ratio through which air is diffusing is less than the overall value.

It is also apparent from the scattering of points obtained in the separation range that the actual phenomenon of separation is a very unstable thing. Repeated attempts to confine the exact region of separation within smaller limits by this method were unsuccessful.

The next step was an attempt by boundary layer ionization to show a large enough change in weight flow before separation so that it might be demonstrated as a positive increase over the above limit as found with unionized air flow.

Numerous electrical difficulties made the above task impossible. The first was that the corona electrode as originally designed would spark over at only slightly above corona onset thus eliminating the desired concentration of positive ions in the boundary


TITLE
PLOT OF PRESSURE RISE EFFICIENCY V.S. WEIGHT FLOW FOR THE DIFFUSER.
FIGURE 10
0.80
0.78
0.76
0.74
0.72
0.70
E
0
0.1
0.2
0.3
W Lbs/SEC

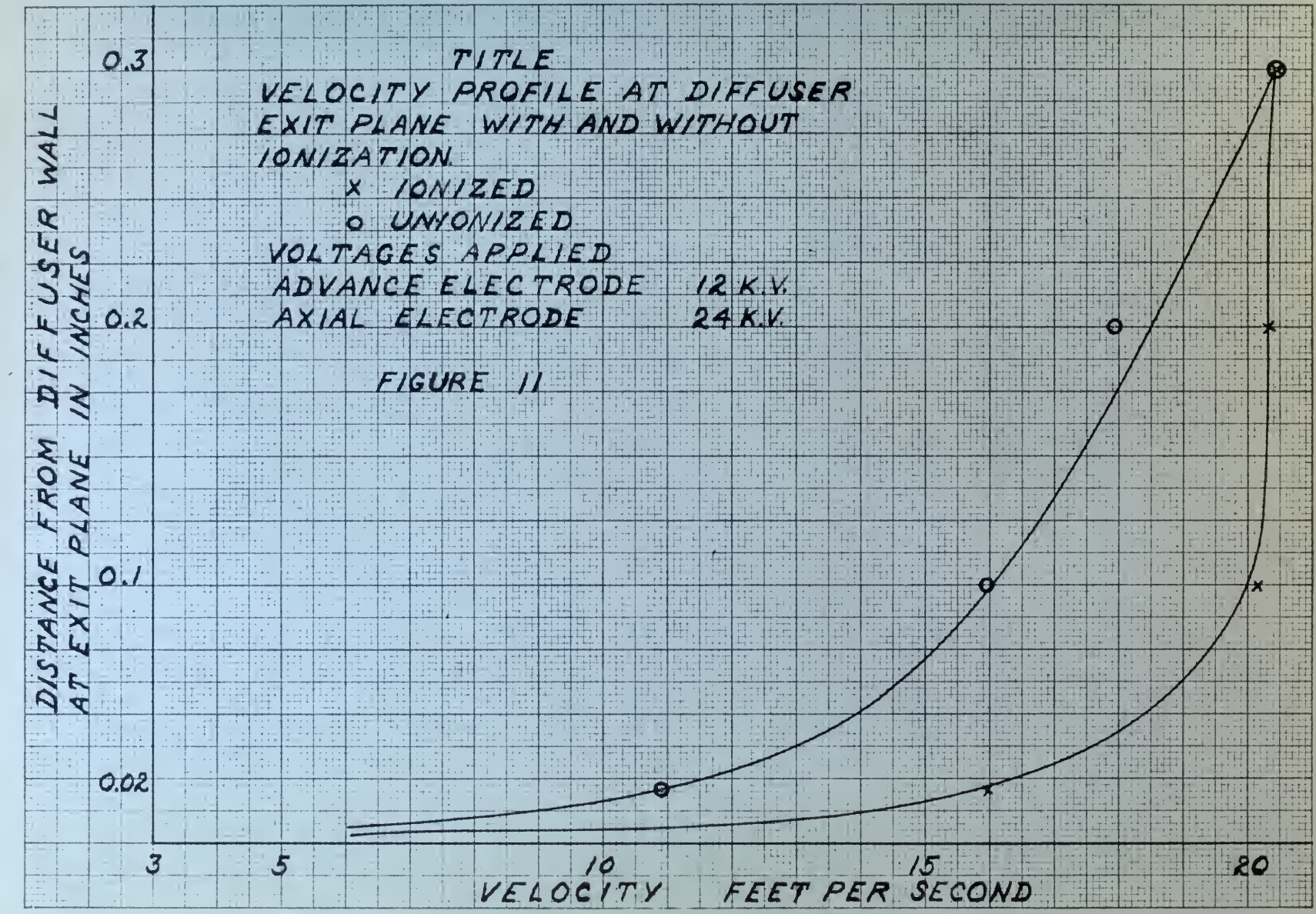
layer. The reason for this was traced to two factors. First of all the original distance of 0.7 inches from the advance electrode to the diffuser was too short. Secondly the diffuser edge which was intended to provide a point of field concentration was not in fact a point of convergence for the lines of force of the electric field. These two defects were remedied by moving the advance electrode 1.69 inches from the diffuser entrance and then placing a second electrode of one thousandth inch shim brass one fourth of an inch upstream from the diffuser entrance and connecting it to ground potential. This arrangement provided two sharp edges at a high potential difference and sufficiently far apart to allow a good corona discharge.

Further tests, however, showed that because of comparatively large quantities of oil droplets present in the air supply from the compressor and the mechanical difficulties of installing proper insulation in the air metering section that the advance electrode voltage must be limited to twelve thousand volts to prevent sparking to ground. This potential provided from seventy to ninety micro-amperes of corona current which means that only a small percentage of the total number of molecules existing in the boundary layer were actually ionized. In view of this fact it was immediately realized that since the number of ions could not be increased linearly with stream velocity because of equipment limitations that the effects desired would be most noticeable and easily measured at the lower velocities.

At the lower rates of flow it was decided that the velocity profile at the diffuser exit plane would be the best criterion of the effectiveness of ionization. For an unionized flow with normal energy losses occurring at the wall the velocity would behave according to well known laws varying from zero at the wall to the stream velocity at the center. With an ionized flow which was one hundred percent effective, that is prevented all molecules from striking the wall and losing all but their thermal energy components, we might expect to find the stream velocity persisting to within one molecular collision radius of the wall. The actual profile observed if ionization reduces boundary layer energy losses appreciably should lie somewhere between these two limits.

To observe experimentally the effect of boundary layer ionization on the velocity profile a small pitot tube and alcohol micromanometer were used to record the dynamic head in inches of alcohol at measured distances from the diffuser wall at the exit plane. The results of these tests with and without ionization are shown on Figure 11 by plotting velocity versus distance from the wall.

These data indicate that as the theory predicts the number of molecular collisions with the wall has been materially reduced. The velocity profile using an ionized boundary layer has been considerably flattened and the free stream velocity has been closely approached at only 0.10 inches from the wall where formerly it existed only beyond 0.30 inches from the wall. At 0.02 inches ionization of


TITLE
VELOCITY PROFILE AT DIFFUSER EXIT PLANE WITH AND WITHOUT IONIZATION
x IONIZED
o UNIONIZED
VOLTAGES APPLIED
ADVANCE ELECTRODE 12 K.V.
AXIAL ELECTRODE 24 K.V.
FIGURE 11
DISTANCE FROM DIFFUSER WALL AT EXIT PLANE IN INCHES
0.3
0.2
0.1
0.02
3
5
10
15
20
VELOCITY FEET PER SECOND

Experimental Data for Determination of Velocity Profile at Diffuser Exit Plane With and Without Ionization

Dynamic Pressure in Inches of Alcohol at Various Distances from Diffuser Wall				Kilovolts on Electrodes	
0.02"	0.10"	0.20"	0.30"	Advance	Axial
0.035	0.075	0.095	0.123	0	0
0.040	0.080	0.105	0.123	12	21
0.050	0.085	0.105	0.123	12	23
0.075	0.120	0.122	0.123	12	24

Velocities in ft./sec. Computed from Above Data				Kilovolts on Electrodes	
0.02"	0.10"	0.20"	0.30"	Advance	Axial
10.91	15.97	17.95	20.45	0	0
15.97	20.19	20.35	20.45	12	24

the flow resulted in a velocity change of from 10.9 feet per second to 15.9 feet per second or an increase of 46%. Out in the free stream at a distance of 0.3 inches from the wall ionization of the boundary layer could not be expected to change the velocity for it had previously been unaffected by frictional losses at the wall.

Section 5: Recommendations for Future Research (43)

Part A: The Mathematical Problem

The problem of elimination of boundary layer energy losses as it has here been attacked is essentially one of momentum transfer from ionized molecules next to the wall to adjacent molecules farther out in the free stream. The momentum transferred must be such that the unionized molecule will just be prevented from collision with the wall and not given any appreciable cross stream velocity component. Accordingly, since the foregoing experimental data has proven the validity of the theory of this mechanism the first and all-important next step must be a careful mathematical analysis of boundary layer flow on the basis of kinetic theory in an effort to determine how much momentum transfer is necessary per unit boundary layer volume at a given pressure. This information will make possible equipment designed to give the greatest energy output with a minimum input.

Part B: Drag Reduction

The method of the corona discharge used to produce ionization is little more than an experimental convenience. We have available powerful energy supplies which may be used for ionization in a number of radioactive materials. Since alpha particles are one of the most effective of the radioactive emissions in producing ionization, their use in place of the advance electrodes has a great appeal. If ions are supplied by this means the only electric field necessary would be on the control electrode which draws only a fraction of the total current necessary. This immediately makes it possible to think of applications to airborne equipment where heavy electrical apparatus is out of the question.

Applying the theory as used in this experiment but obtaining the ionizing energy from a radioactive alpha ray source it may be possible to reduce the viscous drag on the wing surfaces of an aircraft. This has already been attempted with considerable success by Mr. A. de Graffenried of Rensselaer Polytechnic Institute and further research on the subject may make possible a practical application.

It is not at all certain that the idea of ionizing the boundary layer air before it flows over the aerodynamic surfaces is the best approach. Several other methods of attack are possible and should be tried. At present the most promising of these seem to be injection of an ionized boundary layer or coating of the flow areas with radioactive materials to give continuous ionization. Of the two,

the coating of the surface with ionizing materials looks most applicable since in any installation requiring ion injection from some outside source the ionic recombination rate will become a controlling factor.

Part C: Plane Shock Wave Stabilization

If we can accept the present theory of shock wave oscillation as the result of a short circuiting of the high pressure on the down-stream side through a sub-sonic boundary layer then there is some hope of using ionized flow to effect stabilization. If a super-acoustic boundary layer can be created then the shock front could extend to the confining wall and thus eliminate the normal cycle of collapse and reformation. If boundary layer energy loss could be prevented at a sufficiently high efficiency by ionic repulsion at the wall as was done in this experiment this effect might be achieved. An alternative would be to prevent energy loss as efficiently as possible by this method and then at the computed mean shock wave position provide two accelerating electrodes to give the boundary layer a supersonic velocity. If successful this would then lead to the formation of a stable plane shock wave at the desired point.

Appendix A

Experimental Data for Analysis of Flow in the Diffuser

1. All pressures in inches of water except ΔP which is in inches of mercury.

2. Barometer Runs 1-6; 30.088 inches of Hg.

Barometer Runs 7-9; 29.808 inches of Hg.

Run	T_o^o F	ΔP	P_o	P_{Th}	P_1	P_2	P_3	P_4	P_5	Location
1	98.5	15.3	25.2		21.8	11.2	5.2	3.1	1.2	W
				36.4	22.6	12.5	6.2	4.0	2.4	C
2	98.5	11.9	18.8		16.5	8.4	3.8	2.2	0.9	W
				27	17.2	9.5	4.7	3.0	1.8	C
3	98.5	9.4	14.5		12.9	6.5	2.9	1.7	0.7	W
				20.6	13.4	7.3	3.6	2.2	1.2	C
4	98.5	6.7	10.1		9.1	4.5	2.1	2.15	0.5	W
				14.4	9.2	5.1	2.5	1.6	0.95	C
5	98.5	3.1	4.3		4.1	2.0	0.9	0.5	0.27	W
				6.2	4.0	2.3	1.1	0.8	0.5	C
6	98.5	2.0	2.7		2.6	1.3	0.55	0.28	0.21	W
				4.0	2.7	1.55	0.8	0.5	0.4	C
7	98.0	10.7	16.7		14.7	7.2	3.4	2.0	0.75	W
				23.4	14.7	8.3	4.0	2.55	1.50	C
8	105	11.22	17.7		15.7	7.9	3.6	2.15	0.83	W
				24.9	15.6	8.6	4.3	2.6	1.5	C
9	106	10.3	16.1		14.2	7.2	3.3	1.9	0.73	W
				22.6	14.2	7.9	3.8	2.3	1.35	C

Flow Metering Calculation

Run	Gage Pressures P_o in.H_2O	P_o lbs./ft.^{2}	ΔP in.H_g	ΔP lbs./ft.^{2}	$P_M = \Delta P + P_o$ lbs./ft.^{2}	Abs. Pressures P_o abs. lbs./ft.^{2}	P_M abs. lbs./ft.^{2}
1	25.2	130.9	15.3	1081	1211.9	2252.9	3333.9
2	18.8	97.6	11.9	842	939.6	2219.6	3061.6
3	14.5	75.4	9.4	664	739.4	2197.4	2861.4
4	10.1	52.5	6.7	473	525.5	2174.5	2647.5
5	4.3	22.35	3.1	219	241.35	2144.4	2363.4
6	2.7	14.01	2.0	141	155.01	2136.0	2277.0
7	16.7	86.8	10.7	756	842.8	2191.8	2947.8
8	17.7	92.0	11.22	794	886.0	2197.0	2991.0
9	16.1	83.6	10.3	728	811.6	2188.6	3000.2

Run	P_M/P_o	M_M	$f(M_M) = \frac{W \sqrt{T_o}}{P_M A_M}$	$\sqrt{T_o}$	$f(M_M) \cdot P_o \cdot A_M$	W lbs./sec.
1	1.480	0.770	0.746	23.62	5.60	0.237
2	1.382	0.696	0.670	23.62	4.96	0.210
3	1.305	0.630	0.600	23.62	4.39	0.186
4	1.219	0.540	0.512	23.62	3.71	0.157
5	1.100	0.373	0.350	23.62	2.50	0.106
6	1.064	0.300	0.278	23.62	1.98	0.084
7	1.347	0.667	0.640	23.61	4.67	0.198
8	1.362	0.630	0.652	23.79	4.76	0.200
9	1.370	0.687	0.660	23.80	4.81	0.202

Diffuser Efficiency Calculations

Run	P_T gage in. H_2O	P_T gage lbs./ft.^{2}	P_T abs. lbs./ft.^{2}	$\frac{W \sqrt{T_o}}{P_T A_T}$	M_T	$\frac{W \sqrt{T_o}}{P_E A_E}$	M_E
1	36.4	-189.0	1933	0.452	0.480	0.206	0.220
2	27.0	-140.5	1981	0.390	0.415	0.182	0.198
3	20.6	-107.0	2015	0.340	0.365	0.161	0.150
4	14.4	-74.9	2047	0.282	0.305	0.136	0.125
5	6.2	-32.2	2090	0.187	0.201	0.092	0.085
6	4.0	-20.8	2101	0.147	0.155	0.073	0.070
7	23.4	-121.5	1984	0.367	0.390	0.173	0.187
8	24.9	-129.2	1976	0.374	0.400	0.176	0.190
9	22.6	-117.2	1988	0.377	0.402	0.189	0.203

Run	$\frac{P_T}{P_E}$	$1 - \frac{P_T}{P_E}$	$\frac{P_o}{P_T}$	$\frac{P_o}{P_E}$	$\frac{p_T}{P_E}$	$1 - \frac{p_T}{P_E}$	
1	0.912	0.088	1.170	1.035	0.885	0.115	0.765
2	0.934	0.066	1.125	1.028	0.914	0.086	0.768
3	0.949	0.051	1.095	1.017	0.929	0.071	0.719
4	0.965	0.035	1.066	1.013	0.952	0.048	0.729
5	0.985	0.015	1.028	1.007	0.980	0.020	0.750
6	0.990	0.010	1.019	1.006	0.987	0.013	0.770
7	0.942	0.058	1.110	1.025	0.923	0.077	0.754
8	0.939	0.061	1.115	1.026	0.920	0.080	0.763
9	0.944	0.056	1.116	1.028	0.921	0.079	0.709

Sample Flow Metering Calculations

Run #1

Symbols used:

M_M = Mach number in metering nozzle exit

A_M = area of metering nozzle = 0.480 in.^{2} = 0.003335 ft.^{2}

ΔP = pressure difference across metering nozzle

P_M = static pressure upstream of metering nozzle

P_o = static pressure downstream of metering nozzle

P_B = barometric pressure

$$P_M - P_o = \Delta P$$

$$P_M = \Delta P + P_o$$

$$P_o = P_M - \Delta P$$

$$\Delta P = \frac{15.30}{29.92} \times 2116 = 1081 \text{ lbs./ft.}^2 \text{ gage}$$

$$P_o = 25.2 \times 5.197 = 130.9 \text{ lbs./ft.}^2 \text{ gage}$$

$$P_M = 1081 + 130.9 = 1211.9 \text{ lbs./ft.}^2 \text{ gage}$$

$$P_B = \frac{30.088}{29.920} \times 2116 = 2122 \text{ lbs./ft.}^2$$

$$P_o = 130.9 + 2122 = 2252.9 \text{ lbs./ft.}^2 \text{ absolute}$$

$$P_M = 1211.9 + 2122.0 = 3333.9 \text{ lbs./ft.}^2 \text{ absolute}$$

$$P_M/P_o = \frac{3333.9}{2252.9} = 1.48$$

Calculating Weight Flow Through Metering Nozzle:

$$\frac{W\sqrt{T_o}}{P_M A_M} = M_M \sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2} M_M^2\right)}$$

$$M_M = \frac{2}{\gamma-1}\sqrt{\left[\left(\frac{P_M}{P_o}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

$$M_M = 0.770$$

$$\frac{W\sqrt{T_o}}{P_o A_M} = 0.746$$

$$W = \frac{0.746 \times P_o \times A_M}{\sqrt{T_o}}$$

$T_o = 558.5^\circ$ R $\qquad \sqrt{T_o} = 23.62$

$$W = \frac{0.746 \times 2252.9 \times 0.003335}{23.62} = 0.237 \text{ lbs./sec.}$$

Sample Diffuser Efficiency Calculation

Run #1

Symbols used:

P_T = pressure at diffuser throat

P_E = pressure at diffuser exit

p = calculated ideal pressure

M_T = mach number at diffuser throat

M_E = mach number at diffuser exit

A_T = area of diffuser throat

A_E = area of diffuser exit plane

$\mathcal{E}$ = diffuser pressure rise efficiency

$$\mathcal{E} = \frac{P_E - P_T}{p_E - p_T} = \frac{1 - \frac{P_T}{P_E}}{1 - \frac{p_T}{p_E}}$$

$p_E = P_E$ since diffuser is discharging to atmospheric pressure.

and:

$P_E = P_E = 2122$ lbs./ft.^{2} absolute

$A_T = 0.925$ in.^{2} $= 0.00642$ ft.^{2}

$A_E = 1.850$ in.^{2} $= 0.01282$ ft.^{2}

Find M_T:

$$\frac{W\sqrt{T_o}}{P_T A_T} = M_T \sqrt{\frac{\gamma g}{R}\left(1 + \frac{\gamma - 1}{2} M_T^2\right)}$$

$$f(M_T) = \frac{0.237 \times 23.62}{1933 \times 0.00642} = 0.452$$

$$M_T = 0.480$$

Find M_E:

$$f(M_E) = \frac{0.237 \times 23.62}{2122 \times 0.01282} = 0.2055$$

$$M_E = 0.220$$

Find p_T:

Let P_o be pressure where $M = 0$

at M_T $\frac{P_o}{P_T} = 1.170$ at M_E $= \frac{P_o}{P_E} = 1.035$

$$\frac{P_T}{P_E} = \frac{P_o}{P_E}\,\frac{1}{\frac{P_o}{P_T}} = \frac{1.035}{1.170} = 0.885$$

$$\mathcal{E} = \frac{1 - \frac{P_T}{P_E}}{1 - \frac{P_T}{P_E}} = \frac{1 - \frac{1933}{2122}}{1 - 0.885} = \frac{0.088}{0.115}$$

$$\mathcal{E} = 0.765$$

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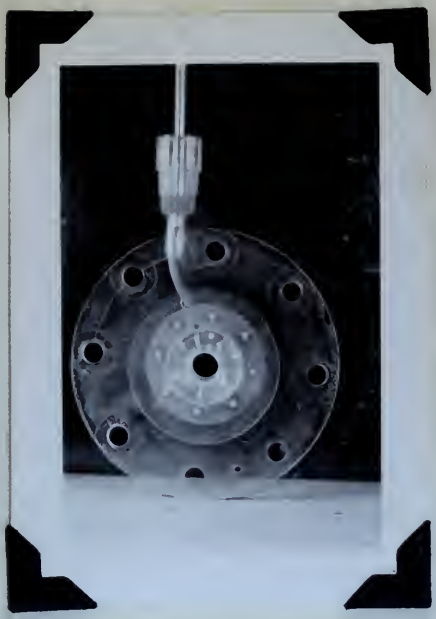
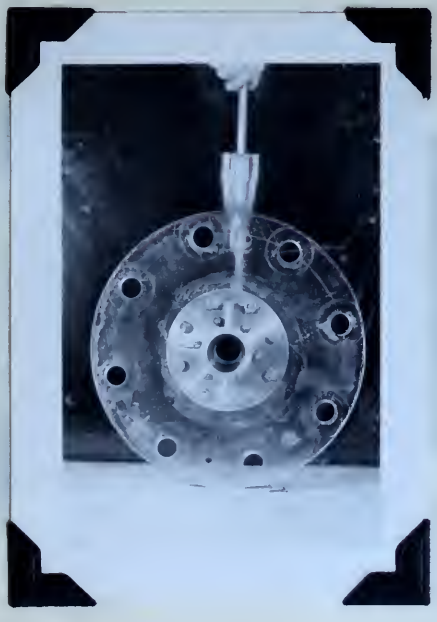
PHOTOGRAPHS OF EQUIPMENT

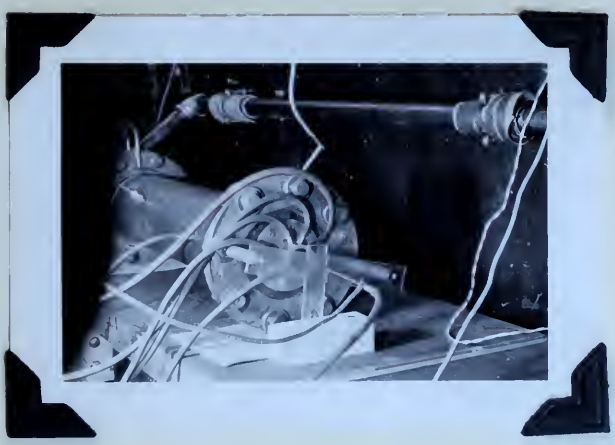


Rear view, showing diffuser entrance and advance electrode voltage input.



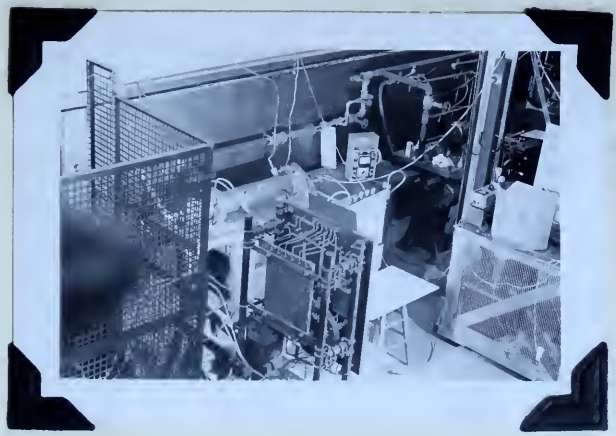
Front view, showing diffuser exit.

Side view of diffuser



View of diffuser after assembly into flow metering unit.

Control board for high voltage supply.



General view of assembled equipment.